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New formulas and predictions for running masses at higher scales in MSSM

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Abstract. The effects of the scale dependent vacuum expectation values (VEVs) on the running masses of quarks and leptons in non-SUSY gauge theories have been considered by a number of authors. Here we use RGEs of the VEVs, and the gauge and Yukawa couplings in the MSSM to analytically derive new one loop formulas for the running masses above the SUSY breaking scale. Some of the masses exhibit a substantially different behaviour with respect to their dependence on the gauge and Yukawa couplings when compared with earlier formulas in the MSSM derived ignoring RGEs of VEVs. In particular, the masses of the first two generations are found to be independent of the Yukawa couplings of the third generation in the small mixing limit. New numerical estimates at the two loop level are also presented.

1 Introduction

One of the important objectives of current research in high energy physics is to understand the masses and mixings of the quarks and leptons in the context of a unified theory of basic interactions. Apart from accounting for the well-known gauge hierarchy problem, the minimal supersymmetric standard model (MSSM) remarkably exhibits the unification of gauge couplings at the SUSY GUT scale, $M_U \simeq 2 \times 10^{16}$ GeV, consistent with the CERN-LEP data [1]. The knowledge of running masses is not only essential near the elctroweak scale, but also near the intermediate and the GUT scales in order to test the models based upon quark–lepton unification [2–10], the Yukawa textures of the fermion masses [3] and the predictive ansatz for neutrino masses and mixings in $SO(10)$ with a unified explanation for all fermion masses [4–7].

The current experimental evidence in favour of atmospheric and solar neutrino oscillations has triggered an outburst of models in particle physics proposing specific fermion mass matrices at high scales with or without supersymmetry. In a number of such models [4–8], it has been found essential to use the running masses of the quarks and charged leptons at the GUT scale as inputs and obtain predictions in the neutrino sector. Running quark masses and the ratio of VEVs of the two Higgs doublets ($\tan\beta$) near the GUT scale also occur in the dim-5 contribution to the proton decay amplitude in SUSY GUTs [9]. More recently, an explanation for the new experimental data on the CP-violating ratio ϵ'/ϵ has been suggested in supergravity/superstring inspired SUSY theories in terms of the running strange quark mass near the Planck scale [8]. In view of the increasing importance of these running masses at high scales, especially for the utilisation in model building in supersymmetric gauge theories, it would be interesting to examine whether improved estimates of these masses exist over those carried out recently [10], while preserving such highly appealing features as the gauge and Yukawa coupling unifications through GUTs [11–24]. The purpose of this paper is to obtain renormalisation group equations, new analytic formulas and numerical estimates of the running masses above the SUSY breaking scale ($M_{\mathrm{S}} \simeq m_t$) in the MSSM including the effects of the running vacuum expectation values of the relevant Higgs scalars.

We follow the renormalisation scheme in which the Yukawa couplings and the VEVs run separately [12,13,19, 20,25]. As the running mass of a fermion is the product of its Yukawa coupling and the corresponding VEV, both runnings influence the analytic formulas and the numerical predictions at higher scales ($\mu > M_Z$) as shown in this paper. On the other hand, it is also possible, in a different scheme, to define the VEVs in such a way that they do not run. For example, Sirlin et al. [26] have expressed the VEV in the SM in terms of physical parameters defined on the mass shell. This makes it possible to avoid separate runnings of the VEVs and Yukawa couplings, but have just the fermion masses directly as running quantities. While it would be quite interesting to examine separately the consequences of such a scheme [26], the present paper addresses the outcome of the renormalisation scheme with runnings of both the Yukawa couplings and the VEVs [19, 20,25]. In Sect. 2, we cite examples where running VEVs

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have been utilised in the non-SUSY standard model (SM) and the two-Higgs doublet model (2HDM) and also state RGEs for VEVs in the MSSM. In Sect. 3 we derive RGEs and analytic formulas. Numerical estimates are reported in Sect. 4. Results are summarised with conclusions in Sect. 5.

2 Running vacuum expectation values and renormalisation group equations

With the demonstration of the possibility of b–τ unification near the non-SUSY $SU(5)$ GUT scale [11], a number of interesting investigations have been made earlier and also recently to examine the behaviour of the running Yukawa couplings as well as the running masses above the electroweak scale. At any scale μ above the electroweak symmetry breaking scale ($\equiv M_Z$), the running mass of a charged fermion 'a' in the SM or MSSM is defined as

$$M_a = Y_a v_a.$$

Here Y_a is a Yukawa coupling and v_a is a VEV. Following [12,13,18–21], we treat both Y_a and v_a to be scale dependent, leading to

$$M_a(\mu) = Y_a(\mu) v_a(\mu). \tag{2.1}$$

Thus, to derive RGE for the running masses $M_a(\mu)$, the RGEs of the Yukawa couplings and running VEVs are essential and they are quite well known both in the non-SUSY and in the SUSY models.

As early as 1979, using certain approximations, Grimus [12] has derived RGEs and analytic formulas for running masses of quarks and charged leptons at higher scales extending up to the non-SUSY GUT scale including RGEs of Yukawa couplings as well as the vacuum expectation value and the latter has been obtained from Higgs scalar wave-function renormalisation in the SM.

In their pioneering work on the infrared fixed point behaviour of the Yukawa couplings, Pendelton and Ross [13] have also utilised the running VEV of the non-SUSY SM Higgs to derive RGEs of the running quark–lepton masses above the electroweak scale extending up to the non-SUSY GUT scale. More recently Balzeleit et al. [18] have utilised the RGE of the VEV of the SM Higgs to derive the RGE for the running masses and the CKM parameters at high scales. They have also examined, numerically, the behaviour of the running VEV at high scales extending up to $\mu \simeq 10^{10}$ GeV. In the non-SUSY two Higgs doublet model (2HDM) the RGEs of the running VEVs have been derived by Cvetic, Hwang and Kim [19]. In addition to investigating the suppression of flavour changing neutral currents in the 2HDM up to the high scale of $\mu \simeq 10^{10}$ GeV, the authors have also obtained the running quark masses $m_i(\mu)$ $(i = t, b, c, s)$ including the scale dependence and RGEs of the VEVs through (2.1). But, to our knowledge, so far the scale dependence and RGEs of VEVs in the MSSM have not been exploited to derive analytic formulas and numerical estimates of the running quark–lepton masses above the electroweak scale, although, as stated above, they have been used in the non-SUSY SM and 2HSM [12,13,19]. In view of the increasing importance at high scale values of the running quark–lepton masses, which serve as inputs to models leading to predictions for neutrino masses and mixings, we utilise the well-known RGEs for Yukawa couplings in the MSSM as well as the VEVs of the up-type (v_u) and down-type (v_d) doublets derived in [20,21],

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} \ln v_{u,d} = \gamma_{v_{u,d}} + \text{two loops},$$

$$\gamma_{v_u} = \frac{3g_1^2}{20} + \frac{3g_2^2}{4} - 3\mathrm{Tr}(Y_U Y_U^\dagger),$$
$$\gamma_{v_d} = \frac{3g_1^2}{20} + \frac{3g_2^2}{4} - \mathrm{Tr}(3Y_D Y_D^\dagger + Y_E Y_E^\dagger). \tag{2.2}$$

Here $t = \ln\mu$ and we take the SUSY breaking scale as the top quark mass ($M_\mathrm{S} \simeq m_t$) and assume the validity of (2.2) for $\mu > m_t$. Details of the refinements of the analytic formulas in SM and 2HDM will be discussed separately.

3 Renormalisation group equations and analytic formulas for running masses

The RGEs for the running Yukawa couplings for $\mu > m_t$, which are essential to obtain formulas for the running masses, are [20–23],

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} Y_U = \left[\mathrm{Tr}(3Y_U Y_U^\dagger) + 3Y_U Y_U^\dagger + Y_D Y_D^\dagger - \sum_i c_i^{(u)} g_i^2\right] Y_U,$$

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} Y_D = \left[\mathrm{Tr}(3Y_U Y_U^\dagger + Y_E Y_E^\dagger) + 3Y_D Y_D^\dagger + Y_U Y_U^\dagger - \sum_i c_i^{(d)} g_i^2\right] Y_D,$$

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} Y_E = \left[\mathrm{Tr}(3Y_D Y_D^\dagger + Y_E Y_E^\dagger) + 3Y_E Y_E^\dagger - \sum_i c_i^{(e)} g_i^2\right] Y_E, \tag{3.1}$$

where $c_i^{(u)} = (13/15, 3, 16/3), c_i^{(d)} = (7/15, 3, 16/3)$ and $c_i^{(e)} = (9/5, 3, 0)$. Two loop contributions are given in [20, 21]. Taking the SUSY breaking scale $M_\mathrm{S} \simeq m_t$ and using (2.1)–(3.1), the RGEs for the mass matrices for $\mu > m_t$ in the broken phase of MSSM are defined as

$$16\pi^2 \left(\frac{\mathrm{d}M_U}{\mathrm{d}t}\right) = (-c_i g_i^2 + 3Y_U Y_U^\dagger + Y_D Y_D^\dagger) M_U,$$

$$16\pi^2\left(\frac{\mathrm{d}M_D}{\mathrm{d}t}\right) = (-c_i' g_i^2 + Y_U Y_U^\dagger + 3Y_D Y_D^\dagger)M_D,$$

$$16\pi^2\left(\frac{\mathrm{d}M_E}{\mathrm{d}t}\right) = (-c_i'' g_i^2 + Y_E Y_E^\dagger)M_E, \tag{3.2}$$

where $c_i = (43/60, 9/4, 16/3), c_i' = (19/60, 9/4, 16/3)$ and $c_i'' = (33/20, 9/4, 0)$. Defining the diagonal mass matrices ($\hat{M}_F$) and the Yukawa matrices ($\hat{Y}_F$) through a biunitary transformation and the CKM matrix (V) as in [9], $\hat{M}_F = L_F^\dagger M_F R_F$, $V = L_U^\dagger L_D$, $\hat{M}_F^2 = L_F^\dagger M_F M_F^\dagger L_F$, $\hat{Y}_F^2 = L_F^\dagger Y_F Y_F^\dagger L_F$, we derive the RGEs for $\hat{M}_F^2$,

$$\begin{aligned}
\frac{\mathrm{d}\hat{M}_U^2}{\mathrm{d}t} &= [\hat{M}_U^2, L_U^\dagger \dot{L}_U] + (1/16\pi^2) \\
&\quad \times(-2c_i g_i^2 \hat{M}_U^2 + 6\hat{Y}_U^2 \hat{M}_U^2 \\
&\quad + V\hat{Y}_D^2 V^\dagger \hat{M}_U^2 + \hat{M}_U^2 V \hat{Y}_D^2 V^\dagger), \\
\frac{\mathrm{d}\hat{M}_D^2}{\mathrm{d}t} &= [\hat{M}_D^2, L_D^\dagger \dot{L}_D] + (1/16\pi^2) \\
&\quad \times(-2c_i' g_i^2 \hat{M}_D^2 + 6\hat{Y}_D^2 \hat{M}_D^2 \\
&\quad + V^\dagger \hat{Y}_U^2 V \hat{M}_D^2 + \hat{M}_D^2 V^\dagger \hat{Y}_U^2 V), \\
\frac{\mathrm{d}\hat{M}_E^2}{\mathrm{d}t} &= [\hat{M}_E^2, L_E^\dagger \dot{L}_E] \\
&\quad + (1/16\pi^2)(-2c_i'' g_i^2 \hat{M}_E^2 + 6\hat{Y}_E^2 \hat{M}_E^2),
\end{aligned} \tag{3.3}$$

where the dot inside the commutator on the RHS denotes the derivative with respect to the variable $t = \ln\mu$. The diagonal elements of $L_F^\dagger \dot{L}_F$, (F=U, D, E) are fixed to be zero in the usual manner [22] through diagonal phase multiplication. The RGEs for the Yukawa and the CKM matrix elements remain the same as before [17,22]. Now using the diagonal elements of both sides of (3.3), the RGEs for the mass eigen-values are obtained by ignoring the Yukawa couplings of the first two generations,

$$16\pi^2(\mathrm{d}m_j/\mathrm{d}t) = [-c_i g_i^2 + |V_{jb}|^2 y_b^2]m_j, \quad j = u, c,$$

$$16\pi^2(\mathrm{d}m_t/\mathrm{d}t) = [-c_i g_i^2 + 3y_t^2 + |V_{tb}|^2 y_b^2]m_t,$$

$$16\pi^2(\mathrm{d}m_j/\mathrm{d}t) = [-c_i' g_i^2 + |V_{tj}|^2 y_t^2]m_j, \quad j = d, s,$$

$$16\pi^2(\mathrm{d}m_b/\mathrm{d}t) = [-c_i' g_i^2 + 3y_b^2 + |V_{tb}|^2 y_t^2]m_b,$$

$$16\pi^2(\mathrm{d}m_j/\mathrm{d}t) = [-c_i'' g_i^2]m_j, \quad j = e, \mu,$$

$$16\pi^2(\mathrm{d}m_\tau/\mathrm{d}t) = [-c_i'' g_i^2 + 3y_\tau^2]m_\tau. \tag{3.4}$$

Integrating these equations and using the corresponding low energy values, the new formulas are obtained in the small mixing limit as

$$m_t(\mu) = m_t(m_t) B_u^{-1} \mathrm{e}^{(3I_t + I_b)},$$

$$m_c(\mu) = m_c(m_c)\eta_c^{-1} B_u^{-1},$$

$$m_u(\mu) = m_u(1\,\mathrm{GeV})\eta_u^{-1} B_u^{-1},$$

$$m_b(\mu) = m_b(m_b)\eta_b^{-1} B_d^{-1} \mathrm{e}^{(I_t + 3I_b)},$$

$$m_i(\mu) = m_i(1\,\mathrm{GeV})\eta_i^{-1} B_d^{-1}, \quad i = d, s,$$

$$m_\tau(\mu) = m_\tau(m_\tau)\eta_\tau^{-1} B_e^{-1} \mathrm{e}^{3I_\tau},$$

$$m_i(\mu) = m_i(m_i)\eta_i^{-1} B_e^{-1}, \quad i = e, \mu, \tag{3.5}$$

where the Yukawa coupling (y_f) integrals are defined as

$$I_f = \frac{1}{16\pi^2}\int_{\ln m_t}^{\ln\mu} y_f^2(t)\mathrm{d}t, \quad f = t, b, \tau,$$

and

$$\begin{aligned}
B_u &= (\alpha_1(\mu)/\alpha_1(m_t))^{43/792}(\alpha_2(\mu)/\alpha_2(m_t))^{9/8} \\
&\quad \times (\alpha_3(\mu)/\alpha_3(m_t))^{-8/9},
\end{aligned}$$

$$\begin{aligned}
B_d &= (\alpha_1(\mu)/\alpha_1(m_t))^{19/792}(\alpha_2(\mu)/\alpha_2(m_t))^{9/8} \\
&\quad \times (\alpha_3(\mu)/\alpha_3(m_t))^{-8/9}.
\end{aligned}$$

$$B_e = (\alpha_1(\mu)/\alpha_1(m_t))^{1/8}(\alpha_2(\mu)/\alpha_2(m_t))^{9/8}. \tag{3.6}$$

The parameters $\eta_\alpha(\alpha = u, c, d, s, b, e, \mu, \tau)$ in (3.5) are well known QCD–QED rescaling factors [22]. For $\tan\beta = v_u/v_d$, the RGE is obtained from the difference of the beta functions, $\gamma_{v_u} - \gamma_{v_d}$, and the values at higher scales are given by the one loop analytic formula

$$\tan\beta(\mu) = \tan\beta(m_t)\mathrm{e}^{(-3I_t + 3I_b + I_\tau)}. \tag{3.7}$$

Our analytic formulas for the running masses given in (3.5) and (3.6) may be compared with earlier formulas which have been stated treating the vacuum expectation values to be scale independent for all values of $\mu > m_t$ [23]

$$m_t(\mu) = m_t(m_t) A_u^{-1} \mathrm{e}^{(6I_t + I_b)},$$

$$m_c(\mu) = m_c(m_c)\eta_c^{-1} A_u^{-1} \mathrm{e}^{3I_t},$$

$$m_u(\mu) = m_u(1\,\mathrm{GeV})\eta_u^{-1} A_u^{-1} \mathrm{e}^{3I_t},$$

$$m_b(\mu) = m_b(m_b)\eta_b^{-1} A_d^{-1} \mathrm{e}^{(I_t + 6I_b + I_\tau)},$$

$$m_i(\mu) = m_i(1\,\mathrm{GeV})\eta_i^{-1} A_d^{-1} \mathrm{e}^{(3I_b + I_\tau)}, \quad i = d, s,$$

$$m_\tau(\mu) = m_\tau(m_\tau)\eta_\tau^{-1} A_e^{-1} \mathrm{e}^{3I_b + I_\tau},$$

$$m_i(\mu) = m_i(m_i)\eta_i^{-1} A_e^{-1} \mathrm{e}^{(3I_b + I_\tau)}, \quad i = e, \mu, \tag{3.8}$$

Table 1. Predictions of masses, VEVs, and $\tan\beta(\mu)$ at two different scales in MSSM for $\tan\beta(m_t) = 10.0$ and other low energy values same as in [10]

Parameter	This analysis $\mu = 10^9$ GeV	[10]	This analysis $\mu = 2\times 10^{16}$ GeV	[10]
$\tan\beta$	7.973	10	6.912	10
v_u (GeV)	142.123	173.130	128.085	173.130
v_d (GeV)	17.815	17.312	18.534	17.312
m_t (GeV)	107.52	149^{+40}_{-26}	73.55	129^{+96}_{-40}
m_c (GeV)	0.3373	$0.427^{+.035}_{-.038}$	0.2003	$0.302^{+.025}_{-.027}$
m_u (MeV)	1.178	$1.470^{+.26}_{-.28}$	0.7059	$1.04^{+.19}_{-.20}$
m_b (GeV)	1.580	$1.60 \pm .06$	1.004	$1.00 \pm .04$
m_s (GeV)	0.0478	$0.0453^{+.0057}_{-.0063}$	0.0292	$0.0265^{+.0033}_{-.0037}$
m_d (MeV)	2.4018	$2.28^{+.29}_{-.32}$	1.4632	$1.33^{+.17}_{-.19}$
m_τ (GeV)	1.5177	$1.4695^{+.0003}_{-.0002}$	1.2566	$1.1714 \pm .0002$
m_μ (MeV)	89.088	$86.217 \pm .00028$	73.6226	$68.59813 \pm .00022$
m_e (MeV)	0.422	0.40850306	0.3487	0.32502032

where

$$A_u = (\alpha_1(\mu)/\alpha_1(m_t))^{13/198} \times (\alpha_2(\mu)/\alpha_2(m_t))^{3/2}(\alpha_3(\mu)/\alpha_3(m_t))^{-8/9},$$

$$A_d = (\alpha_1(\mu)/\alpha_1(m_t))^{7/198} \times (\alpha_2(\mu)/\alpha_2(m_t))^{3/2}(\alpha_3(\mu)/\alpha_3(m_t))^{-8/9},$$

$$A_e = (\alpha_1(\mu)/\alpha_1(m_t))^{3/22}(\alpha_2(\mu)/\alpha_2(m_t))^{3/2}. \quad (3.9)$$

It is clear from (3.5) and (3.6) that, compared with (3.8) and (3.9), the new formulas have very significant differences with respect to their functional dependence on the gauge and Yukawa couplings in all cases. The dependence on the $SU(2)_L \times U(1)_Y$ gauge coupling is weaker than the previous results. The top quark running mass depends on its Yukawa coupling as e^{3I_t} instead of e^{6I_t}. Similarly, the b quark running mass depends upon its Yukawa coupling as e^{3I_b} instead of e^{6I_b}. The b quark Yukawa coupling contribution has vanished from the running τ lepton mass, but the contribution of its own Yukawa coupling has been increased to e^{3I_τ}, instead of e^{I_τ}. Clearly, at the one loop level, the masses of the first two generations are found to be independent of the third generation Yukawa couplings.

4 Numerical estimates

In view of the present results, apart from modifying the analytic formulas, earlier numerical mass predictions including [10], where the μ dependence of the VEVs has been ignored, are to be rescaled by $v_u(\mu)/v_u(m_t)$ for the up quark masses and by $v_d(\mu)/v_d(m_t)$ for the down quark and charged lepton masses. While estimating masses, VEVs, and $\tan\beta$ at higher scales, we have solved all relevant RGEs, including those of the VEVs and $\tan\beta(\mu)$, up to two loops with the same inputs at $\mu = m_t$ as in [10]. We find that the input value of $m_t(m_t) = 171 \pm 12$ GeV gives rise to the perturbative limit $y_t(M_{\mathrm{GUT}}) \le 3.54$ at $\tan\beta(m_t) \ge 1.74^{+.46}_{-.28}$. Due to the running being governed by the corresponding RGE at a two loop level, this limit at the GUT scale turns out to be $\tan\beta(M_{\mathrm{GUT}}) \ge .52^{+.14}_{-.10}$, showing that actual solutions to RGEs permit $\tan\beta(M_{\mathrm{GUT}})(\equiv \tan\bar{\beta}) < 1$ near the perturbative limit of $y_t(M_{\mathrm{GUT}})$. We also observe the saturation of the perturbative limit for the b quark Yukawa couplings ($\bar{y}_b$) for $\tan\beta(m_t) \simeq 61$. We have checked that the one loop analytic solutions agree with the full two loop numerical solutions within 5–7% except near the perturbative limits, where the discrepancy increases further due to larger two loop effects.

In Table 1, we present the predictions for VEVs, $\tan\beta$ and masses at two different scales: $\mu = 10^9$ GeV, and $\mu = 2\times 10^{16}$ GeV for the input $\tan\beta(m_t) = 10$. Our solutions of RGEs yield values of $v_u(\mu)$ very significantly different than the assumed scale independent one, although $v_d(\mu)$ is not very significantly different, for $\tan\beta \approx 10$. This feature leads to quite different up quark masses, the most prominent being $m_t(\mu)$. The running VEV of v_u reduces the central value of $m_t(\mu)$ to nearly 72%(52%) at the intermediate (GUT) scale. Similarly, $m_u(\mu)$ and $m_c(\mu)$ are reduced to 80%(67%) and 79%(66%), respectively, at the intermediate (GUT) scale as compared to [10]. As $v_d(\mu)$ is closer to the assumed scale independent value for $\tan\beta \simeq 10$, all the down quark and the charged lepton masses are closer to the values obtained in [10]. But it is clear that significant differences will appear in these cases also in the larger $\tan\beta$ region.

In Table 2, we present GUT scale predictions of the VEVs, $\tan\beta$ and third generation fermion masses, denoted with overbars, as a function of different input values of $\tan\beta(m_t)$. The GUT scale value of $m_t(M_{\mathrm{GUT}})$ is found to nearly reach a minimum, which is approximately half of its perturbative limiting value, for $\tan\beta \simeq 10$. After this minimum is reached, $m_t(M_{\mathrm{GUT}})$ increases slowly with

Table 2. Predictions of VEVs, $\tan\beta$, and third generation fermion masses at the GUT scale as a function of input values of $\tan\beta(m_t)$ and other input masses same as in [10]. The GUT scale values have been denoted with overbars

$\tan\beta(m_t)$	$\tan\overline{\beta}$	$\overline{v_u}$	$\overline{v_d}$	$\overline{m_t}$	$\overline{m_b}$	$\overline{m_\tau}$
1.75	0.521	48.497	93.078	146.144	1.280	1.253
2	0.963	80.87	83.90	100.962	1.116	1.252
5	3.35	123.13	36.74	75.138	1.008	1.252
10	6.910	128.08	18.54	73.556	1.004	1.256
20	14.18	128.75	9.079	73.92	1.022	1.2739
30	22.1	127.899	5.787	75.321	1.0613	1.3078
40	31.443	126.15	4.0119	77.869	1.1317	1.3662
50	44.476	123.05	2.766	82.768	1.274	1.484
60	80.60	116.08	1.440	98.103	1.820	1.924

increasing $\tan\beta$; but the increase is faster for $\tan\beta \geq 50$. Similarly $m_b(M_{\rm GUT})$ shows more than a 10% increase both for smaller (larger) values of $\tan\beta$ below (above) 2.0(40.0) as compared to its value at $\tan\beta = 10$. Also the numerical solution to the RGE for $\tan\beta(\mu)$ exhibits its GUT scale value ($\tan\bar{\beta}$) to be significantly less than the low energy input except until the input approaches the value of $\tan\beta(m_t) \simeq 61$ corresponding to the saturation of the perturbative limit of $y_b(M_{\rm GUT})$. In this region, the GUT scale value of $\tan\bar{\beta}$ exceeds the corresponding low energy input as shown for the case of $\tan\beta(m_t) = 60$.

5 Summary and conclusion

The vacuum expectation values of Higgs scalars in the non-SUSY SM, 2HDM and MSSM are subject to quantum corrections and their RGEs have been derived using the relevant wave-function renormalisation [18–20]. The running VEVs have been used effectively in the non-SUSY SM by Grimus [12], Pendleton and Ross [13] and Balzeleit et al. [18] and also in the 2HDM by Cvetic et al. [19] to study and estimate the running masses of the quarks and charged leptons and other quantities much above the electroweak scale. In this paper we have obtained RGEs for the running masses and derived new analytic formulas in the MSSM, compared to the earlier formulas in the same model, which were derived by assuming the VEVs to be scale independent [23] for any value of $\mu > m_t$. Our numerical estimates are also found to be different from those of [10], which have been obtained ignoring the scale dependence of the VEVs. When the effects of the running VEVs are included, the one loop formulas for all the running masses exhibit a substantially different functional dependence on the gauge and Yukawa couplings. In particular, in the limit of vanishing CKM mixings, the running masses of the first two generations are found to be independent of the Yukawa couplings of the third generation. This conclusion in the MSSM is similar to that in the SM by Pendleton and Ross [13] who have included the RGE for the VEV to derive formulas for the running masses leading to their conclusion: "despite the appearance of a large Yukawa coupling, the equations for the evolution of light quark masses are not greatly changed". The dependence on the gauge couplings in the new formulas are also different from the earlier ones in the MSSM. We suggest that these improved estimates on the running masses at high scales be used as inputs to test models proposing a unified explanation of the quark–lepton masses including neutrinos [5–7]. For the sake of comparison with the numerical estimates made in [10], where the scale dependence of the VEVs has been ignored, we have presented our new estimates at $\mu = 10^9$ GeV and 2×10^{16} GeV. As our formulas and method of numerical estimation are valid at all scales above the top quark mass as long as the coupling constants remain perturbative, the present procedure can be adopted to estimate the running masses at any scale, $m_t \leq \mu \leq M_{\rm GUT}$. Details of the estimates at other scales and the effect of the variation of the SUSY breaking scale will be examined elsewhere. Combining the present results with those of the low energy formulas for the neutrino masses, derived using the see saw mechanism [24], also shows that the light Majorana neutrino masses of the first two generations are independent of the Yukawa couplings of the third generation.

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